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1995 J. Phys. A: Math. Gen. 28 4669

(http://iopscience.iop.org/0305-4470/28/17/004)

J. Phys. A: Math. Gen. 28 (1995) 4669–4678. Printed in the UK

Non-equilibrium layered lattice gases*

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Received 1 November 1994

Abstract. We have studied several non-equilibrium lattice gases with particle-conserving dynamics. The lattice consists of two planes, and particles interact (attract) only with nearest-neighbour particles within the same plane but may hop to the other. In addition to the standard heat bath at temperature T, a mechanism exists that biases a principal axis, namely, we assume either that particles are also driven by a constant field, or else that exchanges along the given axis occur completely at random as governed by an additional heat bath at infinite temperature. Kinetic mean-field theory and high-temperature series expansions reveal some interesting properties of steady states which we compare with the case of the plane. In particular, the system exhibits at T' the strong phase transition reported for driven gases, and also phase segregation below $T^* < T'$ whose nature varies with a dynamical rule.

1. Introduction and definition of models

The systems of interest entail two identical square lattices *placed back to back* at a distance apart equal to the lattice spacing; they are denoted $\Lambda \equiv \lambda_1 \cup \lambda_2$, where $\lambda_1 \cap \lambda_2 = \emptyset$ and $\lambda = \lambda_1 = \lambda_2 = Z^2$. Let $|\lambda|$ and $|\Lambda| = 2|\lambda|$ represent the volumes. The configurations are $\sigma = \{\sigma_r; r \in \Lambda\}$ and $\sigma^i = \{\sigma_r; r \in \lambda_i\}$, with $\sigma_r = 0, 1$, $i = 1, 2$. Therefore, $\rho \equiv |\Lambda|^{-1} \sum_{r\in\Lambda} \sigma_r$ and $\rho_i \equiv |\lambda_i|^{-1} \sum_{r\in\lambda} \sigma_r$ are densities. The two boxes are uncoupled, i.e. the total configurational energy is $H \wedge (\sigma) = H(\sigma^1) + H(\sigma^2)$, where $H(\sigma) = -4J \sum_{|r-s|=1} \sigma_r\sigma_s$ is the ordinary Ising Hamiltonian; here, $|r - s| = 1$ indicates that r and s are a pair of nearest-neighbour (NN) lattice sites. The equilibrium properties of Λ can be related to those of λ so that they are well understood (Achahbar *et al* 1995a). However, our concern here is on non-equilibrium steady states.

A non-equilibrium condition may be induced in Λ as, for example, in the driven lattice gas (DLG) (cf Garrido *et al* 1990a, Spohn 1991, and references therein). That is, one may assume that the probability of configuration σ at time t satisfies the master equation

$$\partial P(\sigma; t)/\partial t = \sum_{|r-s|=1} [c(\sigma^{rs}; r, s)P(\sigma^{rs}; t) - c(\sigma; r, s)P(\sigma; t)] \qquad (1.1)$$

where σ^{rs} represents σ with the occupation variables at r and s exchanged, and the transition probability per unit time (*rate*) for that exchange is given by

$$c(\sigma; r, s) = \phi[\beta H \wedge (\sigma^{rs}) - \beta H \wedge (\sigma) - E\hat{\imath} \cdot (r - s)(\sigma_r - \sigma_s)]. \qquad (1.2)$$

Here, $\beta = (k_B T)^{-1}$ is the inverse temperature, $|r - s| = 1$, and $\hat{\imath}$ is a unit vector along a given principal direction within the plane (then, $\hat{\jmath}$ and $\hat{k}$ are the unit vectors perpendicular

* Partially supported by the Dirección General de Investigación Científica y Técnica, Project PB91-0709, and Plan Andaluz de Investigación, Junta de Andalucía.

0305-4470/95/174669+10$19.50

to $\hat{\imath}$ that belong to the plane, and points on the other plane, respectively). The function $\phi(X)$ is arbitrary, except that $\phi(X) = e^{-X}\phi(-X)$ so that the problem precisely reduces to the equilibrium problem for $E = 0$. One may interpret that time evolution proceeds by stochastic jumps of particles to NN empty sites, including jumps from one plane to the other, and that a driving field $E\hat{\imath}$, which is constant both in time and in space, adds up to the heat bath at temperature T; consequently, a net steady dissipative current sets in for appropriate boundary conditions. Particle number is conserved during this evolution. The resulting system, to be denoted Λ^E, has been studied by the Monte Carlo (MC) method (Achahbar and Marro 1995) for $E \to \infty$ and $\phi(X) = \min\{1, e^{-X}\}$ (the latter corresponds to the Metropolis algorithm for $E = 0$).

Alternatively, one may produce non-equilibrium steady states in Λ by taking (Garrido *et al* 1990b)

$$c(\sigma; r, s) = \begin{cases} 1 & \text{if } \hat{\imath}\cdot(r-s) = \pm 1 \\ \phi[\beta H \wedge (\sigma^{rs}) - \beta H \wedge (\sigma)] & \text{if } \hat{\imath}\cdot(r-s) = 0 \end{cases} \tag{1.3}$$

for the rate in (1.1) instead of (1.2). Here, $\phi(X) = e^{-X}\phi(-X)$. That is, exchanges occur in this system, to be denoted Λ^T, completely at random (as induced by a heat bath at infinite temperature) along the $\pm\hat{\imath}$ directions and at temperature T otherwise. It is clear that Λ^T has some similarities with Λ^E for $E \to \infty$. In fact, the latter may be interpreted as a particular case of Λ^T lacking symmetry between the $\pm\hat{\imath}$ directions in (1.3); this symmetry sometimes makes Λ^T more convenient for computations. This is the main motivation to study Λ^T. Particle number is also conserved during the time evolution of Λ^T.

We present some analytical results for both Λ^E and Λ^T. This paper is mainly devoted to a study of the former by a kinetic mean-field method and other means for arbitrary values of E. This study has some practical interest. In particular, the consideration of two planes has been shown to be convenient for numerical studies of phase equilibrium (Achahbar *et al* 1995a), and it may also help the understanding of the behaviour of multilayered systems. There is also a more specific motivation for the present study. We confirm below that, as observed first in MC experiments (Achahbar and Marro 1995), Λ^E exhibits a phase transition at a low enough temperature which does not occur in the two-dimensional DLG (nor in the equilibrium case $\Lambda^{E=0}$), and we have studied the influence of varying E and $\phi(X)$ on this phenomenon. The consideration of the other system, Δ^T, has allowed us a high-temperature series expansion in section 3 to enable us to make some conclusions about the nature of spatial correlations. This is probably the first time that these questions have been investigated analytically in layered, *quasi-two-dimensional*, non-equilibrium lattice systems.

2. Non-equilibrium phase transitions

The methods applied before by Dickman (1987, 1988) and Garrido *et al* (1990a, b) to square geometries have been adapted here to study the various phase transitions exhibited by Λ^E and Λ^T. We have been partially guided by MC observations. The MC study of $\Lambda^{E\to\infty}$ had revealed the existence of two distinct transitions, as illustrated in figure 1. For $\rho = 1/2$, $\Lambda^{E\to\infty}$ seems to exhibit a second-order (non-equilibrium) phase transition at T', and a first-order phase transition at $T^* < T'$. Apparently, each plane of $\Lambda^{E\to\infty}$ behaves at T' as an ordinary, two-dimensional DLG at its critical point (this statement is proved below). For $\rho < 1/2$, the situation is similar except that discontinuities also occur at $T'(\rho)$, and both $T'(\rho)$ and $T^*(\rho)$ decrease with decreasing ρ. Moreover, the *liquid* for $\rho < 1/2$ does not completely fill one of the planes at low enough temperature but forms a compact strip which coexists with *gas* filling the rest of this plane and the other plane.

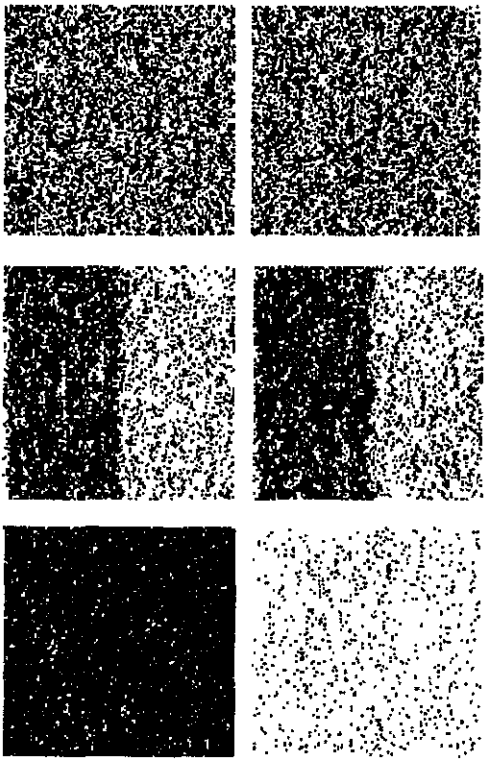

Figure 1. Typical configurations in the two planes of $\Lambda^{E\to\infty}$ from MC experiments (Achahbar and Marro 1995) for $\rho = \frac{1}{2}$ and, from top to bottom, $T > T'$, $T' > T > T^*$, and $T < T^*$, respectively. The field acts vertically.

A comment on some limitations of our method, which are related to the nature of the steady states, is in order. The mean-field method used in part of our study is suited to the study of homogeneous states; it can only give a *mean*, sometimes incorrect description of the states for $T' > T > T^*$, which are expected to be inhomogeneous. In particular, it gives no evidence, for $\rho = 1/2$, of strips such as those in figure 1 but only homogeneous *mean states*. A test of stability under small density gradients reveals that such homogeneity is unstable; however, the same method predicts stable homogeneous states for $T > T'$ (in fact, this is the procedure that we have used to estimate T' below). For $\rho < 1/2$, the method does not correctly describe any of the two different kinds of states with strips that are expected for $T < T'$. In contrast, for $\rho = 1/2$, one may estimate $T^*(< T')$ as the temperature at which ρ_1 and ρ_2 are equal to each other as the system is heated up, starting from homogeneous distributions of different density at each plane.

We also warn of the fact that some correlations between planes are neglected by our kinetic mean-field method. We do not expect this to be a good approximation for $T' > T > T^*$, which seems dominated by interplane currents (Achahbar *et al* 1995b); no attempt to describe this is made here. We are interested in $T > T'$ and $T < T^*$ where the MC study suggests weak interplane correlations. The existence of slowly decaying correlations at high temperature is demonstrated in section 3, but the corresponding amplitude turns out to be sufficiently small to allow for our simplifying assumption.

2.1. Kinetic mean-field theory

The idea here is to reduce (1.1) to a few equations for the time evolution of mean local quantities by neglecting long-wavelength fluctuations and restricting correlations to those between a small number of sites. This is performed in practice by considering a small, compact domain of sites, say D, which allows for the elementary, anisotropic dynamical processes which occur in the system, and suppresses the correlations between the particles within D and those in the rest of the system. With this aim, one neglects the possibility of fluctuations within the surface of D in such a way that the resulting description corresponds to a first-order mean-field if only pair correlations within D are allowed. This is the case we have studied explicitly.

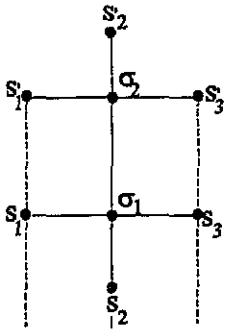

Figure 2. The part, C_1, of the basic domain $D = C_1 \oplus C_2$ lying in plane one. The occupation variables at the *interior* sites (σ_1 and σ_2) and at the *exterior* sites (s_i and s'_i with $i = 1, 2, 3$, as indicated) are also shown. The two possible orientations of this cluster with respect to the applied field are to be considered.

Consider a set, say C_1, in plane one that comprises two *interior* NN sites whose occupation variables are denoted σ_1 and σ_2, and its six *exterior* NNs, as in figure 2. Then, consider a similar set of sites, C_2, placed back to back in plane two, and define the domain $D = C_1 \oplus C_2$. As indicated above, we assumed that, for a given configuration of the system, the probabilities for the configurations of these sets are related: $p(\sigma_D) = p(\sigma_{C_1})p(\sigma_{C_2})$. Furthermore, we write

$$p(\sigma_{C_i}) = p_i(\sigma_1, \sigma_2) \prod_{j=1}^{3} p_i(s_j|\sigma_1) p_i(s'_j|\sigma_2) \qquad p_i(s_j|\sigma_1) = \frac{p_i(s_j, \sigma_1)}{p_i(\sigma_1)} \quad (2.1)$$

where $p_i(s_j, \sigma_1)$ and $p_i(s'_j, \sigma_2)$, $i = 1, 2$, $j = 1, 2, 3$, are the probabilities for the indicated occupation variables within C_i. Assuming (2.1) amounts to neglecting further correlations beyond the pair ones. Finally, consistency requires that we assume that the only dynamical processes are exchanges between any of the four pairs of interior NNs in D. Of course, one needs to consider the two possible different orientations of D with respect to the applied field.

The quantities to be monitored within each plane, $i = 1, 2$, are $p_i(+) = p_i$ (where $+$ stands for $\sigma = 1$), and $p_i(+,+)$ which we denote either $z_i^{\parallel}$ or $z_i^{\perp}$ according to whether the pair $++$ is parallel or perpendicular to $E\hat{\imath}$, and there is conservation of $\rho = 1/2(\rho_1 + \rho_2)$. Therefore, summing over all possible configurations of D leads (after some computer algebra) to five kinetic equations, and one may obtain the corresponding stationary properties by numerical integration. Further details of the method may be found in Garrido *et al* (1990a).

A main conclusion refers to the phase segregation that has been observed for $\rho = \frac{1}{2}$ below T^* (cf figure 1). The present model predicts the existence of a phase transition for any value of E, such that $|\rho_1 - \rho_2| \neq 0$ and $z_1^{\parallel} \neq z_2^{\parallel}$, $z_1^{\perp} \neq z_2^{\perp}$ for $T < T^*(E)$, and $\rho_1 = \rho_2$ and $z_1^{\parallel} = z_2^{\parallel}$, $z_1^{\perp} = z_2^{\perp}$ for $T > T^*(E)$ (cf figure 3). The asymmetry shown by the planes below $T^*(E)$ corresponds to the MC observation that condensation occurs in one of the planes only. $T^*(E)$ decreases with increasing E, with a well-defined limit for $E \to \infty$, and $T^*(0)$ equals the Bethe (equilibrium) critical temperature for the square Ising model, as expected.

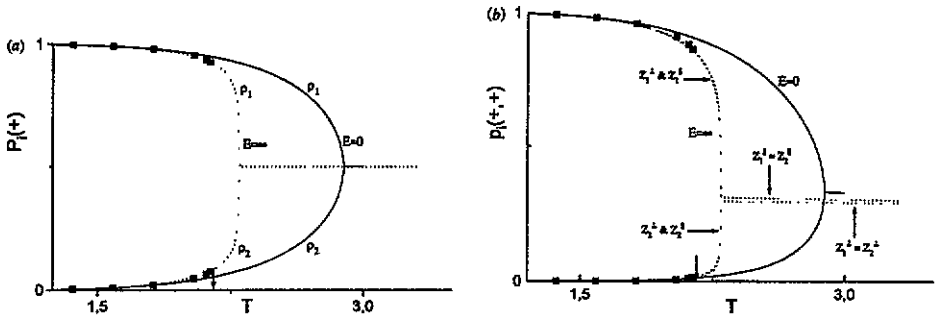

Figure 3. Results from kinetic mean-field equations for the states at low enough temperature locating $T^*(E)$ for $\rho = \frac{1}{2}$. Full squares correspond to Monte Carlo data from Achahbar and Marro (1995), and the temperature locating the corresponding transition of first order is indicated by an arrow. (*a*) The density within each plane, $p_i(+) = \rho_i$, as a function of temperature, for $E = 0$ (equilibrium) and $E = \infty$, as indicated. (*b*) The same for the quantity $p_i(+,+)$, which is a measure of the energy, denoted either $z_i^{\parallel}$ or $z_i^{\perp}$ according to the direction of the bond $(++)$ with respect to the field.

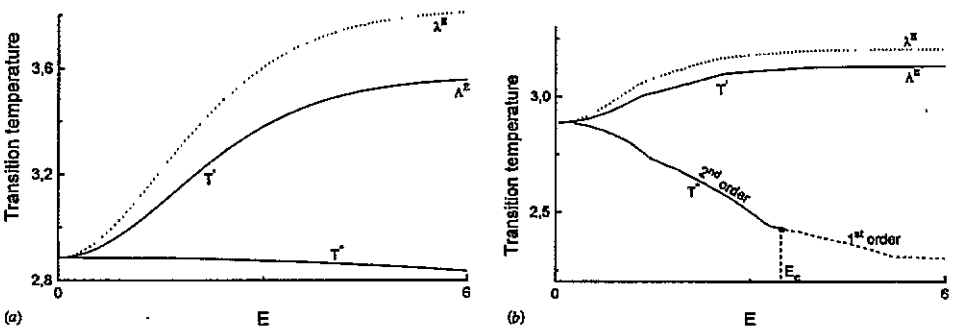

Figure 4. The field dependence of the transition temperatures T' and T^*, for $\rho = \frac{1}{2}$, and for (*a*) the Kawasaki rate, $\phi(X) = 1/(1 + e^X)$, and (*b*) the Metropolis rate, $\phi(X) = \min\{1, e^{-X}\}$. The latter induces the existence of a non-equilibrium tricritical point at E_c separating second- from first-order phase transitions. The function $T'(E)$ for the ordinary, two-dimensional DLG obtained within the same approximation is also shown (curves labelled λ^E).

A more detailed study indicates the essential role played by the *rate*, i.e. the function ϕ in (1.2). For instance, while the phase transition is of second-order for any E for the *Kawasaki rate*, namely, $\phi(X) = 1/(1 + e^X)$ (figure 4(*a*)), the *Metropolis rate* $\phi(X) = \min\{1, e^{-X}\}$ (figure 4(*b*)) induces the existence of a *tricritical point* at $E_c = 3.5 \pm 0.1$. The latter separates a region of critical points for $E < E_c$ from a region of first-order phase transitions for $E > E_c$, in agreement with the MC observation for $E \to \infty$. Furthermore, we find that

$T^*(E)$ decreases with E more markedly for the Metropolis than for the Kawasaki rates; cf table 1 for a comparison of critical temperatures for saturating fields. The singular behaviour exhibited by $T^*(E)$ at a few points in figure 4(*b*) is a peculiarity of the function $\phi(X)$ in this case.

Table 1. Transition temperatures (in units of the corresponding equilibrium critical temperature) for $\rho = 1/2$ and saturating field, for different systems and methods: MCM, Monte Carlo simulation with Metropolis rate (from Achahbar *et al* (1995a, b)); MFM, present mean-field theory for Metropolis rate; and MFK, (the latter for Kawasaki rate).

	$\Lambda^{E\to\infty}$			$\lambda^{E\to\infty}$		
	MCM	MFM	MFK	MCM	MFM	MFK
T'	1.30	1.09	1.23	1.38	1.11	1.32
T^*	0.95	0.80	0.99	—	—	—

Our method cannot describe $T' > T > T^*(E)$ well; it predicts a homogeneous disordered state above $T^*(E)$ in which $\rho = \rho_1 = \rho_2$, $z_1^{\parallel} = z_2^{\parallel}$, and $z_1^{\perp} = z_2^{\perp}$.

2.2. *Stability of the high-temperature phase*

We have also studied the high-temperature phase transition which occurs for $\rho = \frac{1}{2}$ at T' (cf figure 1) by examining the response of the homogeneous solutions obtained by applying the preceding method to a small density gradient. Given our assumption about the lack of interplane correlations for $T > T'$, it reduces to the study of one plane (which is characterized by $\rho = \frac{1}{2}$, $z^{\parallel}$ and $z^{\perp}$), as done previously by Dickman (1988) and Garrido *et al* (1990a), except that particles may now hop to the other plane. The essentials of the method are as follows.

First, one makes the assumption that the transition at T' is (for $\rho = \frac{1}{2}$) of second-order, as observed in MC experiments for $E \to \infty$. Next, one modifies the known solution by introducing a density gradient $\nabla\rho$ along $\hat{\jmath}$, i.e. transverse to the field. This transforms the state $(\rho = \frac{1}{2}, z^{\parallel}, z^{\perp})$ into $(\rho', z^{\parallel\prime}, z^{\perp\prime})$. This may be performed in practice as follows. Consider the cluster in figure 2 with the field acting vertically. Then, for pairs such as (s_1, s_1') that are within the row ℓ parallel to the field, one takes $p(+,+) = z^{\parallel} - (5 - 2\ell)\nabla\rho$ and $p(-,-) = z^{\parallel} + (5 - 2\ell)\nabla\rho$, while $p(+,-) = p(-,+) = \frac{1}{2} - z^{\parallel}$ remains unchanged; for pairs that are perpendicular to the field within the column k, one takes instead $p(+,+) = z^{\perp} - 2(2 - k)\nabla\rho$, $p(-,-) = z^{\perp} + 2(2 - k)\nabla\rho$, $p(-,+) = \frac{1}{2} - z^{\parallel} - \nabla\rho$, and $p(-,+) = \frac{1}{2} - z^{\parallel} + \nabla\rho$. This amounts to introducing a gradient $2\nabla\rho$; $\nabla\rho$ is typically taken equal to 10^{-6}. The next step is to compute the new probabilities for $p(\rho_D)$ as indicated in (2.1). Then, the transversal particle current,

$$J_{\perp} = \Sigma_{\sigma D}(\sigma_s - \sigma_s)c(\sigma; r, s)p(\sigma_D) \qquad \hat{\imath}\cdot(r - s) = 0 \tag{2.2}$$

is estimated for the transformed state. If $J_{\perp}$ tends to cancel out $\nabla\rho$, we interpret that the original homogeneous state is stable, i.e. the given solution corresponds to $T > T'$; otherwise, the solution is assigned in general to the region $T^* < T < T'$ where strips form. The case $J_{\perp} = 0$ is expected to correspond to $T'(E)$.

This method produces the values for $T'(E)$ reported in figure 4 for two different rates; in particular, it is observed that $T'(E)$ increases with E. Alternatively, figure 4 includes a comparison between the present case of Λ^E with two planes and the ordinary two-dimensional DLG (denoted λ^E here). As first observed in MC experiments for $E \to \infty$

(Achahbar *et al* 1995b), the function $T'(E)$ differs slightly from one case to the other, i.e. $T'(E)$ remains smaller than the corresponding critical temperature for the plane for any $E > 0$. This interesting result is discussed further in the next subsection.

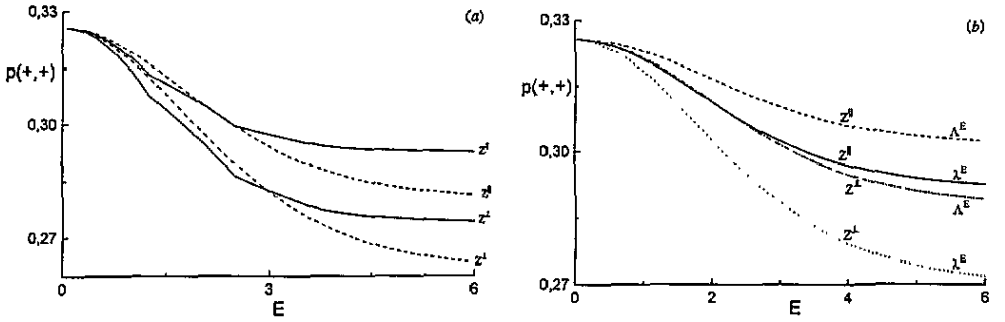

Figure 5. The field dependence at temperature $T = 3.2$ of the function $p(+,+)$ for pairs parallel ($z^{\parallel}$) and perpendicular ($z^{\perp}$) to the field, as indicated: (*a*) for the Metropolis rate (full curve) and for the Kawasaki rate (broken curve), and (*b*) a comparison for the Kawasaki rate between the cases of one (λ^E) and two (Λ^E) planes.

Figure 5(*a*) illustrates the behaviour at high temperature with E of $z^{\parallel}$ and $z^{\perp}$ for the two rates considered; it is observed, in particular, that $z^{\parallel} > z^{\perp}$, as expected on simple grounds. Figure 5(*b*) compares, for a given rate, the behaviour of $p(+,+)$ for Λ^E and λ^E. The curves for λ^E in figure 5 clearly illustrate the role of the field in producing anisotropies, and the comparison between the two cases in figure 5(*b*) illustrates some essential differences. That is, $p(+,+)$ is smaller for any E for Λ^E than for λ^E, and $z^{\parallel}$ and $z^{\perp}$ are closer to each other for Λ^E. In other words, the possibility of the hopping of particles to the other plane tends to decrease anisotropies.

2.3. Λ^E and λ^E compared

The origin of the observed differences in $T'(E)$ and $p(+,+)$ between the case of one and two planes deserves a comment. Let us consider a domain D consisting of five sites within each plane of Λ^E such that the central ones are NNs. That is, $\sigma_D = \{\sigma_1, s_j; \sigma_2, s'_j;\ j = 1,2,3,4\}$, where σ_1 and σ_2 correspond to NN sites each at different plane, and s_j and s'_j are the occupation variables corresponding to NN sites of σ_1 and σ_2, respectively. The time variation of any quantity, say μ, due (only) to interplane exchanges is

$$\left(\frac{\partial \mu}{\partial t}\right)_{\text{interplanes}} = \sum_{\{\sigma_i\}} \Delta\mu p(\sigma_D)\phi(\Delta H) \tag{2.3}$$

where $\Delta\mu$ and $\Delta H = 4\beta J(\sigma_1 - \sigma_2)\Sigma_j(s_j - s'_j)$ denote the associated variations of μ and energy $H \wedge (\sigma)$. One expects that, in so far as one considers disordered states at high enough temperature, the two planes are equivalent, and the interplane exchanges cannot modify isotropy within each plane; i.e. $z = z_1^{\parallel} = z_1^{\perp} = z_2^{\parallel} = z_2^{\perp}$. Consequently, we have to deal with two independent equations. One of them simply corresponds to conservation of $\rho = 1/2$ by interplane exchanges. The other is

$$\left(\frac{\partial z}{\partial t}\right)_{\text{interplanes}} = 2\sum_{n=0}^{4} f(n)n[z(\rho - z)]^{4-n}\phi(-4\beta Jn)[(\rho - z)^{2n} - z^2 e^{-4\beta Jn}]. \tag{2.4}$$

The only solution of this equation is $z = \rho(1 + e^{-2\beta J})^{-1}$ which is the equilibrium solution for $\lambda^{E=0}$ within the present approximation.

Summing up, $\partial\mu/\partial t$ is for Λ^E the sum of (2.3) and two more terms that describe the particle–hole exchanges within each plane, and only (2.3) is independent of E. The stationary solution for one plane cancels out the plane terms for any $E \geqslant 0$, while it causes (2.3) to vanish only for $E = 0$. Therefore, $\partial\mu/\partial t = 0$ for $t \to \infty$ due to cancellations between the three terms, and any quantity μ is different whether one considers Λ^E or λ^E for any $E > 0$.

3. Non-equilibrium correlations

The method in this section consists of performing a high-temperature series expansion of the master equation, as in Garrido *et al* (1990b), where we refer for further details.

One has from (1.1) that

$$\partial G_n(r)/\partial t = \sum_{\{|s-s'|=1; s\in D, s'\notin D\}} \langle \sigma_D(\sigma_s\sigma_{s'} - 1)c(\sigma; s, s')\rangle. \tag{3.1}$$

Here, $G_n(r) = \langle\sigma_D\rangle$, $\sigma_D \equiv \prod_{r\in D}\sigma_r$, D now represents any domain of n sites in Λ, and $\langle\cdots\rangle$ is the average with respect to $P(\sigma; t)$. The order parameter $m(r) = \langle\sigma_r\rangle$ and the pair correlation function $g(r) = G_2(r)$ correspond to $D \equiv \{r\}$ and $D \equiv \{\mathbf{0}, r\}$, respectively; we write $r = (x, \ell)$ hereafter, where $\ell = 1, 2$ refers to the plane of Λ, and $x = (x, y)$ is the location within the plane. Then, some of the steady-state properties of systems such as Λ^T defined for the rate (1.3) may be obtained by making a series expansion around $\beta = 0$. For example, if one has $\phi(\beta\Delta H) = 1 - \beta\Delta H + \cdots$ for the rate function, where $\Delta H \equiv H \wedge (\sigma^{rs}) - H \wedge (\sigma)$, one may formally write that $\langle\cdots\rangle \simeq \langle\cdots\rangle_{\beta=0} + \beta\langle\cdots\rangle_1$ at high enough temperature. In particular, it follows that

$$g(r) = \langle\sigma_{(0,1)}\sigma_{(x,\ell)}\rangle = m^2 + \beta g^{(1)}(\mathbf{0}, 1; x, \ell) + O(\beta^2) \qquad (x, \ell) \neq (0, 1) \tag{3.2}$$

where the Fourier transform of $g^{(1)}(\mathbf{0}, 1; x, \ell)$ for $\ell = 1$ and 2, namely, $\hat{g}_{\ell',\ell}(k) \equiv \Sigma_x e^{ik\cdot x} g^{(1)}(\mathbf{0}, \ell'; x, \ell)$, is

$$\hat{g}_{1,1}(k) = [(5 - \alpha)\omega(k; q) + g^{(1)}(\mathbf{0}, 1; \mathbf{0}, 2) - q\alpha]/(6 - \alpha) \tag{3.3a}$$

and

$$\hat{g}_{1,2}(k) = [\omega(k; q) - g^{(1)}(\mathbf{0}, 1; \mathbf{0}, 2) + q\alpha]/(6 - \alpha) \tag{3.3b}$$

respectively (which reduces to $\omega(k; q)$ for λ, i.e. for the case of only one plane). Here, $\alpha = \alpha(k) \equiv 2[\cos(k_x) + \cos(k_y)]$, and

$$\omega(k; q) \equiv [q_1 - q_2\cos(k_x) - q_3\cos(k_y) + q\{\cos(k_x + k_y) + \cos(k_x - k_y) + \cos(2k_y)\}]/(\alpha - 2) \tag{3.4}$$

with $q \equiv (1 - m^2)^2$, $q_1 \equiv 2g^{(1)}(\mathbf{0}, 1; \hat{\imath}, 1) + 2g^{(1)}(\mathbf{0}, 1; \hat{\jmath}, 1)$, $q_2 \equiv 2q + g^{(1)}(\mathbf{0}, 1; \hat{\imath}, 1)$ and $q_3 \equiv q + g^{(1)}(\mathbf{0}, 1; \hat{\jmath}, 1)$. Therefore, one has at high enough temperature, assuming good convergence of (3.2), that the dominant contribution to the spatial correlations in Λ^T is

$$g^{(1)}(\mathbf{0}, 1; x, \ell) \simeq (a_\ell x^2 - b_\ell y^2)(x^2 + y^2)^2 \tag{3.5}$$

where the coefficients a_ℓ and b_ℓ depend on ℓ, i.e. the behaviour differs, but only quantitatively for intraplane and interplane correlations.

This spatial decay of correlations according to a power law (instead of the more familiar exponential law) has been found previously for the plane λ^T (Garrido *et al* 1990b) using a method similar to that given above. Alternatively, two Langevin equations with a drift term

have been studied that correspond to anisotropic systems with conserved and non-conserved order parameters, respectively; it has been shown (Grinstein 1991, Grinstein *et al* 1993) that a linear coupling between them preserves the power-law behaviour, while this transforms into the exponential law for nonlinear coupling (as would correspond, for instance, to the DLG in section 1 if one adds sufficient *spin flips* to the ordinary particle–hole exchanges). It is not clear to us whether these results apply to Λ^E and Λ^T given that the total density is conserved in these cases. Consequently, it is an interesting fact that (3.5) is valid for any two points in Λ. It follows, in particular, that two points at different planes are also correlated at high temperature. This differs, essentially, from the corresponding situation at equilibrium (e.g. for $E = 0$ in Λ^E) in which case the layered system behaves precisely as the Onsager's square lattice (Achahbar *et al* 1995a).

For completeness, we refer also to a variation of λ^T. That is, the lattice is a d-dimensional simple *cubic* and dynamics consists of a combination of exchanges with probability $1 - p$ and spin flips with probability p. The former are defined in (1.3), i.e. they occur completely at random along $\pm\hat{\imath}$, and at temperature T otherwise. The flips occur with rate $p\phi[\beta H \wedge (\sigma^r) - \beta H \wedge (\sigma)]$ where σ^r denotes σ with σ_r changed to $-\sigma_r$. This case reduces to λ^T as $p \to 0$ (and $d = 2$). Then, using the same method as above, one finds at high temperature that

$$g(r) \propto r^{1/2(1-d)} \exp(-r\Gamma^{1/2})[a_0 + r^{-1}(a_1 x^2 - a_2|y|^2)]. \tag{3.6}$$

Here, $\Gamma \equiv 2p(1-p)^{-1}$, $r \equiv |r|$, $r = (x, y)$, where x denotes the ordinate along $\hat{\imath}$, $y = (y_1, y_2, \ldots, y_{d-1})$, and a_0, a_1 and a_2 are constant. This interesting result probably applies to the DLG variation studied by Binder and Wang (1989).

4. Conclusion

Various quantities have been monitored for the two-dimensional DLG, λ, and its layered, *quasi-two-dimensional* variation, Λ, within a mean-field approximation. We have observed the crossover from equilibrium to non-equilibrium behaviour as the applied electric field E is increased. In particular, we have described how the saturating condition (no particle may jump against the field, *as if* $E \to \infty$), which is often simulated in the computer, is reached. This is interesting because laboratory experiments are reported to refer to *small* fields. Alternatively, we have confirmed the MC observation (for $E \to \infty$) that a novel phase transition is exhibited by Λ at $T^*(E)$ which is below the more familiar critical temperature $T'(E)$. The nature of this transition has been shown to vary with both the dynamical rule and the value of E. It follows that $T^*(E)$ decreases with increasing E, while $T'(E)$ increases with E. This is probably related to the existence of interplane currents relating the two strips that occur for $T' < T < T^*$; however, this phenomenon cannot be investigated by the present method.

Further comparison between λ and Λ has indicated that its steady states differ for any $E > 0$. In particular, $T'(E)$ is larger for λ for any value of $E > 0$, as first observed in MC experiments for $\rho > 0.2$ and $E \to \infty$. That is, the existence of an additional plane makes the DLG less anisotropic. The same is apparent in figure 5(*b*), for instance. It seems related to a difference between the dynamical rules for the two systems: the rule for λ is a competition between the field $E\hat{\imath}$ and the thermal process along $\hat{\jmath}$, while Λ involves an extra thermal randomness along $\hat{k}$ that tends to partially compensate the field anisotropic effects.

Consideration of the anisotropic two-temperature model has allowed us to investigate spatial correlations in layered systems. The extra degree of freedom that characterizes Λ

relieves, in a sense, the condition of constant particle density (which does not need to be conserved within each plane). However, we find for Λ the same kind of slow, power-law spatial relaxation found for λ. This implies, in particular, that, unlike in equilibrium, the two planes of Λ are strongly correlated due to thermal interplane exchanges in spite of the fact that any bond between the planes is broken.

Finally, we have studied a d-dimensional lattice in which one adds, with probability p, spin flips at temperature T to the exchanges (1.3) that occur with probability $1 - p$. That is, particle number is not conserved in this case. We find that power-law correlations are exponentially modulated for any $p \neq 0$.

References

Achahbar A, Garrido P L and Marro J 1995a unpublished
Achahbar A, Garrido P L, Marro J and Alonso J J 1995b unpublished
Achahbar A and Marro J 1995 *J. Stat. Phys.* **78** 1493
Binder K and Wang J S 1989 *J. Stat. Phys.* **55** 87
Dickman R 1987 *Phys. Lett.* **122A** 463
——1988 *Phys. Rev.* A **38** 2588
Garrido P L, Lebowitz J L, Maes C and Spohn H 1990b *Phys. Rev.* A **42** 1954
Garrido P L, Marro J and Dickman R 1990a *Ann. Phys., NY* **199** 366
Grinstein G 1991 *J. Appl. Phys.* **69** 5441
Grinstein G, Jayaprakash C and Socolar J E S 1993 *Phys. Rev.* E **48** R643
Spohn H 1991 *Large Scale Dynamics of Interacting Particles* (Berlin: Springer)